Trajectory Formation in Step-tracking Wrist Movement

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Abstract—
Step-tracking wrist movement has been a good test bed for computation model of neural motor control. In this paper, we apply our motor control model to the center-out step-tracking wrist movement and show that our model can predict spatio-temporal patterns in movement kinematics. In addition, we discuss the mechanism that determines the characteristic of the movement kinematics.

Keywords—Motor Control, Step-tracking Wrist Movement, Trajectory Formation

1 Introduction
Since Hoffman and Strick [1] had made an intensive study of the muscle activation pattern during step-tracking wrist movement, the movement has been a good test bed to investigate how the CNS controls a set of redundant muscles. From the standpoint of computational motor neuroscience, several optimal control models were applied to the movement to reproduce experimentally observed muscle activation patterns [2, 3, 4]. However, it has not been clear whether those models can also predict the spatio-temporal patterns of movement kinematics.

In addition to the muscle activation patterns, some characteristic features of the movement kinematics can be seen in the experiment of Hoffman and Strick [1]. In their experiment, the joint angles of the wrist was projected on the plane composed of two orthogonal axes representing radial/ulnar deviation and flexion/extension. The path of joint angles became almost straight when the target of the movement was located on either of the two axes. Meanwhile, when the target was located between the two axes, the path curved toward the axis of radial/ulnar deviation. Furthermore, the peak speeds were reported to be larger during the movements toward radial/ulnar deviation than flexion/extension. In this paper, we apply our motor control model [5] to the center-out step-tracking wrist movement task and show that our motor control model can predict the experimentally observed patterns of movement kinematics.

2 Methods
2.1 Musculoskeletal system of the wrist
The wrist is modelled as a 2-DOF (flexion/extension and radial/ulnar deviation) mass-spring-dumper linear system actuated by five muscles, ECRL, ECRB, ECU, FCU, and FCR. The joint dynamics of the wrist along each of horizontal axis (i = 1) and vertical axis (i = 2) is described as

\[ \tau_i = M_i \ddot{\theta}_i + B_i \dot{\theta}_i + K_i \theta_i \]  

(1)

where \( \tau_i, \theta_i, M_i, B_i, \) and \( K_i \) denote torque, angle, inertia, viscosity, and elasticity of ith direction, respectively. Note that \( \theta_1 \) and \( \theta_2 \) represent different joint rotations in the three different forearm posture, fully pronated and supinated postures, and their midway posture. For example, positive value of \( \theta_1 \) is assigned to ulnar deviation in pronated posture and it is assigned to extension in the midway posture. The tension of \( j \)th muscle is determined by the elastic and viscous terms, and rest length depending on its activation level \( a_j \).

\[ T_j(\theta, \dot{\theta}, a_j) = (k_j^0 + k_j^1 a_j) (l_j^0 + l_j^1 a_j - \sum_{i=1}^{2} d_{ji} \theta_i) - (b_j^0 + b_j^1 a_j) (\sum_{i=1}^{2} d_{ji} \dot{\theta}_i) \]  

(2)

Here, \( (k_j^0, k_j^1), (b_j^0, b_j^1), \) and \( (l_j^0, l_j^1) \) are the parameter sets for muscle elasticity, viscosity, and stretch length. Meanwhile, \( d_{ji} \) is the \((j,i)\) element of moment arm matrix \( D \), and represents moment arm of \( j \)th muscle against ith axis. Its value is determined by the muscle’s pulling direction \( \phi_j \) as \( d_{j_1}, d_{j_2} = 0.015 \times [\sin\phi_j, \cos\phi_j] \). Note that the pulling direction of each muscle changes as the forearm posture changes (figure 1). We determined their values in accordance with the simulation of [2]. The relationship between muscle activation level and motor command signal is described as a second order linear system with time constants of 92.6 ms and 60.5 ms. Finally, joint torques are determined as the sum of the products of muscle tensions and moment arm.

\[ \tau_i = \sum_{j=1}^{5} d_{ji} T_j \]  

(3)

2.2 Motor control model

Figure 2 illustrates architecture of the motor control model that we proposed [5]. Our model consists of three modules, ISM (inverse statics model), FBC (feedback controller), and FDM (forward dynamics model). Total motor command signal sent to the wrist muscles corresponds to a sum of outputs of the ISM and FBC. The ISM generates a feedforward command...
signal that shifts the wrist’s equilibrium to the final target position. On the other hand, the FDM predicts future state of the wrist given the current state and the outgoing motor command signal sent to the wrist muscles. The FBC generate feedback command signal according to the deviation between final target and predicted future state.

### 3 Results

Figure 3 shows the joint angle paths during step-tracking movements toward 12 targets radially located around neutral position \( ([\theta_1, \theta_2] = [0, 0]) \). When the forearm posture is in the midway posture, movement trajectories showed similarity with the experimental data of Hoffman and Strick [1]. We can see that the joint angle paths became almost straight against the four targets on the flexion/extension and radial/ulnar directions. On the other hand, the movements against the other targets exhibited curved paths. When the movements included some degree of radial deviation, the paths curved toward radial deviation. On the contrary, the path curved toward ulnar deviation during the movements including some degree of ulnar deviation. In the midway posture, peak speeds in the four movements with flexion, extension, radial deviation and ulnar deviation were 152°/s, 150°/s, 189°/s, and 187°/s, respectively.

### 4 Discussion

In this paper, we showed that our model can predict characteristic features of movement kinematics observed in the experiment of Hoffman and Strick [1]. In our simulation, two factors seemed to play important roles in the generation of directionally dependent trajectory curvature and movement speed. Those are non-uniform intervals of the five pulling directions (figure 1), and utilization of the ISM and FBC for motor control (figure 2). The stiffness matrix \( \hat{K} \) caused by the elasticity of the muscles is obtained as follows.

\[
\hat{K} = C^T \text{diag}(\{k_1^0 + k_1^a a_i, \cdots \}) D
\]  

Since the non-uniformity of the pulling directions makes \( C^T D \) anisotropic, \( \hat{K} \) also becomes anisotropic. With the same reason, the viscosity matrix \( \hat{\lambda} \) caused by the viscosity of the muscles becomes anisotropic. Now, when the equilibrium of the wrist is shifted to the target position \( \theta^d \) by the ISM, movement of the wrist can be described by following equation.

\[
M\ddot{\theta} = -(B + \hat{\lambda})\dot{\theta} - (K + \hat{K})(\theta - \theta^d)
\]

Here, \( M = \text{diag}([M_1, M_2]) \), \( B = \text{diag}([B_1, B_2]) \), and \( K = \text{diag}([K_1, K_2]) \). At the beginning of the movement (\( \theta = \dot{\theta} = 0 \)), the direction of the acceleration \( \ddot{\theta} \) is rotated toward the long axis of a stiffness ellipse with respect to \( \hat{K} \). This makes the joint angle path curves when the target is not on long or short axis of the stiffness ellipse. In addition, the size of \( \theta \) becomes larger and movement becomes faster as \( \theta^d \) gets closer to the long axis of the stiffness ellipse. In figure 1, stiffness ellipses with respect to \( \hat{K} \) in three forearm postures are plotted. In the case of midway posture, we can see that the long axis of the stiffness ellipse almost overlap with the radial/ulnar axis. As the result, joint angle paths curved toward radial/ulnar axis and peak speed got larger in the movements with radial and ulnar deviation. Note that the gain matrix of the FBC also becomes anisotropic due to the non-uniformity of the pulling directions. As the result, the shape and orientation of the stiffness ellipse with respect to the feedback gain become almost same as those of the stiffness ellipse with respect to the muscle elasticity. This causes same effect on the movement trajectory as the case of the ISM.

In figure 3, we also plotted simulated joint angle paths in fully supinated and pronated postures. As in the case of the midway posture, the paths tend to curve toward long axis of the stiffness ellipse shown in figure 1. Note that the paths in the pronated posture became almost straight because that the stiffness ellipse is close to a circle in that posture. It is our future work to test whether the simulated trajectories in supinated and pronated posture match the experimental data.

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### References


