Emergence of multiple continuous attractors in coupled neuronal oscillators by inclusion of three-body interactions

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Abstract—We demonstrate that networks of neuronal oscillators with three-body interactions can store multiple phase-lag patterns as continuous attractors, along which the degree of synchrony varies continuously. Such a mechanism could greatly enhance the encoding and retrieval of analog information in cortical neuronal networks.

Keywords—Continuous attractors, Three-body interactions, Associative memory, Neuronal oscillators.

1 Introduction

Rhythmicity, which is suggested to play a crucial role in neuronal functions like cognition and memory, is an essential property of neuronal networks in both the cortex and in many subcortical structures like the thalamus, hypothalamus, hippocampus, and spinal cord. In the hippocampus and cortex, endogenous rhythms are critical for encoding information. Indeed, the interaction of multiple oscillatory networks may be a general mechanism for the representation of information in the brain [1]. Several theoretical studies have demonstrated that coupled neuronal oscillators can store phase-lag patterns as discrete attractors [2] (Fig. 1a). These studies assumed that signal transmission from one neuron to another is mediated exclusively by unidirectional synaptic communication (a two-body interaction). Oscillating networks with only two-body interactions fall into a number of stable states. However, neurons communicate through multiple pathways, and these signaling mechanisms are distributed in both space and time. One of the most important potential neural encoding mechanism is heterosynaptic plasticity, in which synaptic signal transmission is modulated by the activities of other neurons (a three-body interaction) or complex dendritic interactions. A recent theoretical study investigating oscillatory networks revealed that three-body interactions allow the network to acquire an infinite number of attractors, wherein the degree of synchrony among oscillators can vary continuously depending on initial conditions [3].

In the present study, we demonstrate that networks with three-body interactions can store multiple phase patterns as continuous attractors. Furthermore, the degree of synchrony with respect to the stored pattern varies continuously along continuous attractors (Fig. 1b). Continuous attractors may allow neuronal networks to encode analog information [4]. The aims of this study are to explore the mechanisms of encoding and storage of analog information in oscillator networks exhibiting these complex attractor structures, and to discuss the possible roles of three-body interactions in brain function.

2 Model

We consider a network of phase oscillators,

\[
\frac{d\phi_i}{dt} = \omega_i + \Gamma_i(\phi_1, \ldots, \phi_N) \quad (i = 1, 2, \ldots, N),
\]

where \(\Gamma_i\) is the interaction term, and \(\phi_i(t)\) and \(\omega_i\) are the phase and natural frequency of the \(i\)th neuron, respectively. Although each oscillator may represent a set of neurons or a single neuron that exhibits time-periodic activity (rhythmicity), we refer to each oscillator as a neuron for simplicity. Although neurons generally interact through synaptic connections, according to the phase-reduction argument [5], we can assume a simple form of interaction that is periodic and depends only on the phase difference between pre- and postsynaptic neurons. This relationship can be expressed as \(\Gamma_i = \sum_{j=1}^{N} \Gamma_{ij}(\phi_i, \phi_j). \quad \Gamma_{ij}(\phi_i, \phi_j) = J_{ij} \sin(\phi_j - \phi_i + \beta_{ij}).\) Using this conventional two-body interaction, the network can store phase-lag patterns...
as discrete attractors, wherein the energy is locally minimum (Fig. 1a). To consider more complex synaptic interactions, we adopt a simple form of three-body interactions (equation 2) and thus

$$\frac{d\phi_i}{dt} = \omega_i + \sum_{j,k} J_{ij}\sin(\phi_j - \phi_i + \beta_{ij})J_{ik}\cos(\phi_k - \phi_i + \beta_{ik}).$$

(2)

The three-body interaction is determined by the coupling strength $J_{ij}$ and delay parameter $\beta_{ij}$. These parameters equal the absolute value ($J_{ij}$) and argument ($\beta_{ij}$) of a complex synaptic coefficient $C_{ij}$. The complex synaptic coefficient $C_{ij}$ as determined by the Hebbian learning rule is

$$C_{ij} = J_{ij}\exp(i\beta_{ij}) = \frac{\sqrt{2K}}{N}\sum_{j,\mu} e^{i(\theta_{ij}^\mu - \theta_i^\mu)},$$

(3)

where $\theta_i^\mu$ represents the $\mu$th stored phase pattern for the neuron $i$.

3 Results and discussion

First, we examine a network storing two different random phase patterns. After an initial transient period, the network exhibits several stationary states depending on the initial condition. Here, we define an overlap parameter, $R_\mu(t) \equiv \left| \frac{1}{N} \sum_j e^{i(\phi_j(t) - \theta_i^\mu)} \right|$, to quantify the degree of synchrony with respect to the phase-lag pattern $\mu$; the value is 1 for completely synchronized states and 0 for completely desynchronized states. When an initial state $\{\phi_i(0)\}$ is close to the stored pattern $\{\theta_i^1\}$, the corresponding overlap $R_1$ converges to unity while the other overlap $R_2$ converges to zero; that is, the network completely retrieves pattern 1 (Figs. 2a,2b). Starting from initial conditions distinct from the stored pattern, overlaps can also converge to intermediate values between 0 and 1 (Figs. 2b2,2b3). Note that these states are not transient but equilibrium states. Thus, the network can maintain states with an intermediate degree of synchrony, suggesting that this property can be used to encode and preserve analog information, which is essential to some cortical functions. Partial retrieval states with intermediate overlaps are also observed even when the number of stored patterns is increased and the natural frequencies are slightly distributed (Fig. 2c).

There are still many aspects of three-body model behavior that remain unanswered. Do more realistic neuronal network models also retrieve phase-lag patterns with intermediate overlaps? How stable is a continuous attractor under noise? How are analog sensory signals transformed into overlaps? How precisely can the synchrony be controlled by modulating the network activity with external stimuli? Answering these questions will lead to a better understanding of the role of network dynamics in cognition and memory.

References


